Relaxation Solution of High-Subsonic Cascade Flows

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Theme

THE widely studied problem of compressible flow through a cascade of airfoils has been solved numerically for subcritical freestream Mach numbers by applying a relaxation technique to the governing equations for steady, irrotational, isentropic flow. The technique of solution and the numerical results, which agree reasonably well with a set of experimental pressure distributions, are of interest because they demonstrate a successful application of a steady-state technique to a problem of perpetual interest in the field of turbomachinery, flow through a cascade. The present method, closely related to the nozzle and airfoil solutions of Emmons[1] may be contrasted to the more sophisticated and highly developed time-marching solutions,[2] which predict the flow field as the asymptotic state of an unsteady flow. Though not a direct extention of Emmons' method, the current technique is similar to the widely known work of Katsanis[3] but was developed independently. The two methods differ in numerical procedure and application of the boundary conditions.

Contents

The goal of the present work was to predict numerically the entire compressible flowfield through a cascade of blades. Limitations of the analysis included the assumptions that the flow was two-dimensional, steady, inviscid, adiabatic, and irrotational. The gas was assumed calorically perfect (constant specific heats). The assumption of irrotationality coupled with conservation of energy produced the two equations which were solved numerically, namely

$$\tilde{\rho}\nabla^2\psi - \underset{\sim}{\nabla}\psi\cdot\underset{\sim}{\nabla}\tilde{\rho} = 0 \tag{1}$$

and

$$\left[1+\frac{\gamma-1}{2}M_\infty^2\right]\tilde{\rho}^2-\tilde{\rho}^{\gamma+1}=\frac{\gamma-1}{2}M_\infty^2(\underset{\sim}{\nabla}\psi)^2 \tag{2}$$

Here ψ is the stream function defined by $\tilde{\rho}\tilde{u}=\partial\psi/\partial y$ and $\tilde{\rho}\tilde{v}=-\partial\psi/\partial x$, where x and y are the cartesian coordinates, parallel and normal to the chord line, respectively; $\tilde{u}$ and $\tilde{v}$ are the x and y velocity components normalized by the freestream velocity; $\tilde{\rho}$ is the density normalized by its freestream value; $\underset{\sim}{\nabla}$ is the gradient vector operator; γ is the specific heat ratio; and M_∞ is the freestream Mach number. These two equations were solved simultaneously subject to the following boundary conditions: 1) the inlet flow was assumed uniform and parallel at a given angle a finite distance upstream of the leading edge of a blade (about a chord length); 2) the outlet flow was also assumed uniform a finite distance downstream of the trailing edge (also about a chord length), and either an outlet angle was specified or the Kutta condition was enforced at the trailing edge of the blade; 3) the flow was required to be periodic in the plane of the cascade (parallel to a plane through the leading edges); and 4) wall tangency was enforced by requiring $\underset{\sim}{\nabla}\psi$ to be dependent on the local slope of the blade surface.

The relaxation method used to solve the differential equations and enforce the boundary conditions was based on the familiar second degree, central finite difference scheme and employed the use of residuals rather than iteration. The difference operator assumed a rectangular grid whose relative dimensions depended upon the stagger angle of the cascade. Adjustment of a linear initial guess at the ψ distribution was obtained by successively applying this operator to adjacent grid points so as to sweep the residuals toward the flow boundaries. Enforcement of the inlet and outlet boundary conditions was obtained by 1) repeatedly redistributing the ψ distributions there according to a least squares linear fit to the ψ distributions at some point in the solution, and 2) by locally modifying the difference operator to compute $\underset{\sim}{\nabla}\psi$ from the given flow angles. Similarly, the Kutta condition, when used, was applied by requiring $\underset{\sim}{\nabla}\psi\equiv 0$ in a locally modified difference operator. The residual sweeping procedure was halted when the average of the residual magnitudes reached its apparent asymptote. With a grid structure consisting of 43 x grid points by 11 y grid points (473 points total) convergence was obtained within 400 sweeping cycles in marching about 0.1 in M_∞.

With this programed procedure, solutions were obtained for a cascade of double-convex circular arc airfoils (10% thick) for which the Pratt & Whitney Aircraft Company kindly provided an excellent set of experimental pressure distributions for a particular inlet angle and for freestream Mach numbers ranging from low subsonic to the transonic regime. Figures 1-4 present some of these experimental data along with numerical solutions for an angle of attack of 8°. Over the range of Mach numbers for which the computer program was able to produce solutions, agreement with experiment is generally good. The solution also agrees well with the generally accepted incompressible Douglas-Newmann method shown in Fig. 1. Notice that the greatest disagreement occurs on the lower surface (pressure surface) of the blade and near the trailing edge. While the latter is expected because the numerical solution neglects losses in stagnation pressure, the former is somewhat surprising because the pressure gradient on the suction surface is more favorable to a growing and eventually separating boundary layer; whereas on the lower surface the pressure gradient opposes boundary-layer growth. It is not clear whether this discrepancy can be attributed to experimental difficulties or to theoretical problems in the

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Index categories: Subsonic and Transonic Flow; Airbreathing Propulsion, Subsonic and Supersonic.

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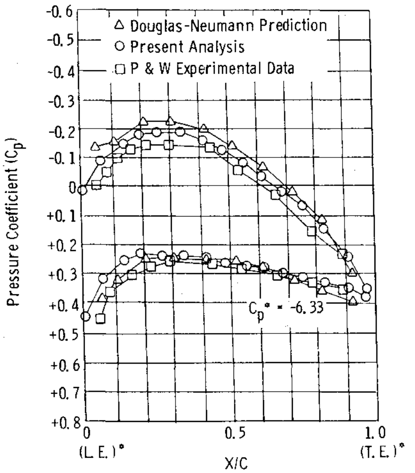

Fig. 1 Pressure distributions for $M_\infty = 0.313$ and $\alpha = 8°$, stagger angle (λ) = 55°, space/chord ratio (S/C) = 0.833 (pressure coefficient is based on inlet dynamic head, superscript * indicates sonic conditions).

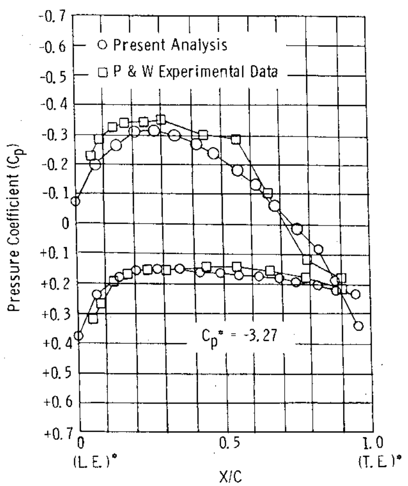

Fig. 2 Pressure distributions for $M_\infty = 0.420$ and $\alpha = 8°$, $\lambda = 55°$, $S/C = 0.833$.

numerical solution. However, the specified experimental angle of attack (10°) was called into question when the usually reliable Douglas-Neumann prediction for 10° disagreed significantly with the experimental data whereas for 8° the excellent agreement of Fig. 1 was shown. It is conceivable that acceleration within the blade row induced by boundary-layer growth on the tunnel wall could produce such disagreement if the angle of attack were also in error. To summarize then, the present version of solving compressible, subcritical cascade flows via the relaxation technique appears to produce quan-

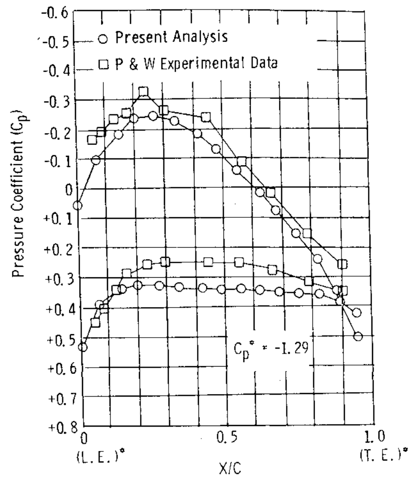

Fig. 3 Pressure distributions for $M_\infty = 0.610$ and $\alpha = 8°$, $\lambda = 55°$, $S/C = 0.833$.

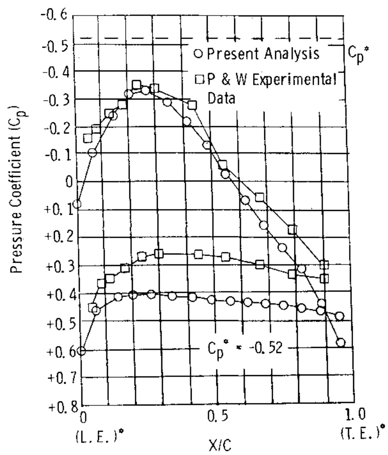

Fig. 4 Pressure distributions for $M_\infty = 0.772$ and $\alpha = 8°$, $\lambda = 55°$, $S/C = 0.833$.

titatively useful results which agree reasonably well with experimental data.

References

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